LO.a: Describe and interpret the binomial option valuation model and its component terms.

1. The multi-period binomial model can be used to value:
   A. Both path-dependent and path-independent options.
   B. path-independent options only.
   C. path-dependent options only.

2. According to the no-arbitrage approach, an investor can synthetically replicate a long call option by:
   A. short selling the underling and using a portion of the proceeds to buy put options.
   B. short selling the underlying and lending a portion of the proceeds.
   C. buying the underlying with partial financing.

3. According to the no-arbitrage approach, an investor can synthetically replicate a long put option by:
   A. short selling the underling and using a portion of the proceeds to buy put options.
   B. short selling the underlying and lending a portion of the proceeds.
   C. buying the underlying with partial financing.

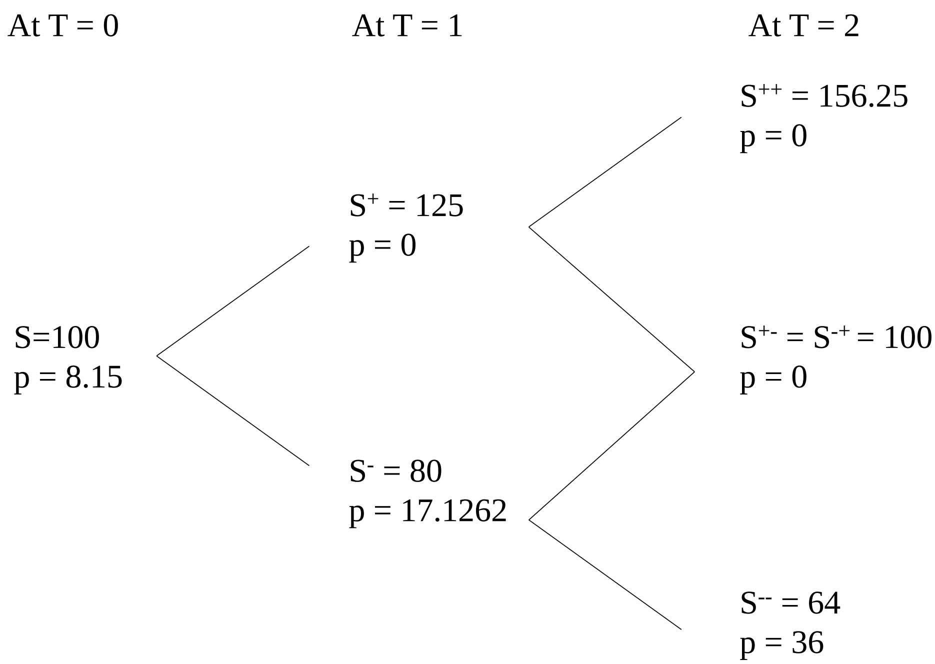
4. Consider a one-year put option with a strike price of $100. The underlying stock is currently trading at $100 and does not pay any dividends. At the end of one year the stock price will either be at $125 or $69. The periodically compounded risk-free interest rate = 5.0%. Assuming a single-period binomial option valuation model, the hedge ratio is *closest* to:
   A. 0.55.
   B. -0.55.
   C. 0.65.

LO.b: Calculate the no-arbitrage values of European and American options using a two-period binomial model.

5. A non-dividend paying stock is trading at €100. A European call option on this stock has two years to mature. The periodically compounded risk-free interest rate is 3%, the exercise price of the option is €100. The up factor is 1.25, and the down factor is 0.80. The risk-neutral probability of an up move is 0.51. The call option value is *closest* to:
   A. €13.80.
   B. €14.20.
   C. €14.50.

6. A non-dividend paying stock is trading at €100. The risk-free rate is 3.00%. A two-year European call option with a strike price of €100 is trading for €14.00. Using put-call parity, value of a two-year European put option with a strike price of €100 is *closest* to:
   A. €9.00.
   B. €8.26.
   C. €10.0.

7. Suppose you are given the following information: S_0 = €100, X = €100, u = 1.25, d = 0.80, n = 2 (time steps), r = 3.00% (per period). The stock is not expected to pay dividends. The risk-neutral probability is 0.51. The tree below shows that price of a two-period European-style option should be 8.15.



The early exercise premium for a similar American style put option is *closest* to:
A. 2.30.
B. 1.40.
C. 0.40.

LO.c: Identify an arbitrage opportunity involving options and describe the related arbitrage.

8. A non-dividend-paying stock is currently trading at $50. A European call option has one year to mature, the periodically compounded risk-free interest rate is 7%, and the exercise price is $50. Assume this option can be priced using a single-period binomial option valuation model, where u = 1.25 and d = 0.80. The market price of the option is $8. Determine whether there is an arbitrage opportunity and if so, how can this opportunity be exploited?
A. There is no arbitrage opportunity.
B. An arbitrage profit can be made by selling options for $8 and buying underlying shares.
C. An arbitrage profit can be made by buying options for $8 and selling underlying shares.

LO.d: Describe how interest rate options are valued using a two-period binomial model.

9. The underlying instrument for interest rate options is *most likely* the:
A. exercise rate.
B. spot rate.
C. futures rate.

10. Which of the following statements is *correct*?
    A. Interest rate options' valuation follows the expectations approach.
    B. A put option on interest rates will be in the money when the spot rate is above the exercise rate.
    C. A call option on interest rates will be in the money when the spot rate is below the exercise rate.

LO.e: Calculate and interpret the values of an interest rate option using a two-period binomial model.

Table 1: Two-Year Binomial Interest Rate Lattice by Year

T = 0	T = 1	T = 2
		Rate = 3.9670% c^{++} = 0.00717
	Value = 0.9626 Rate = 3.8900% c^{+} = ?	
Value = 0.9705 Rate = 3.0380% c = ?		Rate = 3.2450% c^{+-} = 0.0
	Value = 0.9750 Rate = 2.5600% c^{-} = ?	
		Rate = 2.2600% c^{--} = 0.0

11. An analyst is valuing two-year European-style call options on the periodically compounded one-year spot interest rate. Assume the notional amount of the options is €1, the call exercise rate is 3.25% of par, and the RN probability is 50%. Using Table 1, the values of c^{+} and c^{-} at T = 1 are *closest* to:
    A. c^{+} = 0.0035, c^{-} = 0.0000.
    B. c^{+} = 0.0005, c^{-} = 0.0040.
    C. c^{+} = 0.0050, c^{-} = 0.0006.

12. Using Table 2, the value of a call option at T = 0 with €1,000,000 notional principal is *closest* to:
    A. €4,000.
    B. €3,000.
    C. €1,700.

LO.f: Describe how the value of a European option can be analyzed as the present value of the option's expected payoff at expiration.

13. Which of the following statements is *most likely* true? The value of a call option can be calculated as the present value of the expected terminal option payoffs where the discount rate is:
    A. the required return for the underlying stock and the expected payoff is based on the risk neutral probability.
    B. the risk-free rate and the expectation is based on the risk neutral probability.
    C. the required return for the underlying stock and the expected future cash flows are based on the actual probability of the underlying stock going up or down in value.

LO.g: Identify assumptions of the Black–Scholes–Merton option valuation model.

14. Which of the following is *not* an assumption of the BSM model?
    A. The underlying follows geometric Brownian motion.
    B. The underlying has a constant volatility.
    C. Short-selling of the underlying is not allowed.

LO.h: Interpret the components of the Black–Scholes–Merton model as applied to call options in terms of a leveraged position in the underlying.

15. Call option value based on the BSM model is given as:
    A. the bond component minus the stock component.
    B. the stock component minus the bond component.
    C. the stock component plus the bond component.

16. Which of the following statements is *least accurate*?
    A. A put option can be interpreted as lending that is partially financed with a short position in shares.
    B. A call option be interpreted as short-selling the underlying stock and using the proceeds to buy zero-coupon bonds.
    C. A call option can be interpreted as a leveraged position in the underlying stock.

LO.i: Describe how the Black–Scholes–Merton model is used to value European options on equities and currencies.

17. Suppose a stock is trading on the Singapore Stock Exchange at S$60. A portfolio manager believes that the stock price will rise in the next three months and decides to buy three-month call options with exercise price at 62. The risk-free government securities are trading at 1.74%, and the stock is yielding S$ 0.35%. The stock volatility is 30%. Which of the following statements regarding the application of the BSM model to value calls is correct? The BSM model inputs (underlying, exercise, expiration, risk-free rate, dividend yield, and volatility) are
    A. 60, 0.35%, 1.74%, 0.25, 62, 0.30.
    B. 60, 62, 0.25, 0.0174, 0.0035, 0.30.
    C. 62, 0.25, 0.0035, 0.0174, 0.30, 60.

18. A Pakistani importer has to pay fixed euro (€) amounts each quarter for goods. The spot price of the currency pair is 117.60 PKR/€. If the exchange rate rises to 120 PKR/€ then euro would have strengthened because it would take more rupees to buy one euro. The importer feels that the rupee will depreciate in the following months. Hence, he considers buying an at-the-money spot euro call option to protect against this rise. The Pakistani risk-free rate is 6.00% and the European risk-free rate is 1.00%. What is the underlying price, the risk-free rate and the carry rate to use in the BSM model to get the euro call option value?
   A. 117.60, 6.00%, 1.00%.
   B. 1/117.60, 1.00%, 0.00%.
   C. 117.60, 1.00%, 6.00%.

LO.j: Describe how the Black model is used to value European options of futures.

19. The FTSE 100 Index (a spot index) is presently at 6,690 and the 0.25 expiration futures contract is trading at 6,702. Suppose further that the exercise price is 6,690, the continuously compounded risk-free rate is 0.16%, time to expiration is 0.25, and the dividend yield is 4.0%. Based on this information and volatility, $N(d_1) = 0.526$, $N(d_2) = 0.488$. The statement that is *most accurate* under the Black model to value a European call option on the futures contract is:
   A. The call value is the present value of the difference between the futures price of 6,702 times 0.526 and the exercise price of 6,690 times 0.488.
   B. The call value is the present value of the difference between the futures price of 6,702 times 0.526 and the underlying price times 0.474.
   C. The call value is the present value of the difference between the exercise price of 6,690 and the futures price of 6,702.

LO.k: Describe how the Black model is used to value European interest rate options and European swaptions.

20. For an interest rate call option on three-month Libor with one year to expiration, the FRA that expires in one year is 1.20%, the current three-month Libor is 0.84% and the call exercise rate is 0.90%. In applying the Black model to value this interest rate call option, the underlying rate is:
   A. 0.84%.
   B. 0.90%.
   C. 1.20%.

21. A receiver swaption value can be interpreted as:
   A. swap component minus the bond component.
   B. bond component minus the swap component.
   C. bond component plus the swap component.

LO.l: Interpret each of the option Greeks.

22. Which of the following statements is *incorrect*?

A. Delta of an option gives the change in the option value for a given small change in the value of stock, holding everything else constant.
B. All else constant, option gamma is the change in a given option delta for a given small change in stock value.
C. Vega of an option is always negative since an increase in volatility reduces both put and call values.

LO.m: Describe how a delta hedge is executed.

23. Which of the following statements regarding delta hedging of an option is *incorrect*?
    A. A delta neutral portfolio means that portfolio delta is set to zero.
    B. A portfolio of put options with a delta of -1,000, can be hedged by selling 1,000 shares of the underlying stock.
    C. The optimal number of hedging units = - Portfolio delta divided by the delta of the hedging instrument.

24. A portfolio delta is 2,500. This portfolio needs to be hedged with call options. The call options have a delta of 0.5. A delta neutral portfolio is *most likely* attained by:
    A. buying 10,000 call options.
    B. selling 5,000 call options.
    C. selling 2,500 call options.

LO.n: Describe the role of gamma risk in options trading.

25. Which of the following statements is *most likely* correct?
    A. If a portfolio is delta hedged there is no gamma risk.
    B. Gamma has the smallest value when an option is at the money.
    C. Gamma risk arises if there is an abrupt change in the price of the underlying.

LO.o: Define implied volatility and explain how it is used in options trading.

26. Which of the following statements is *least likely* incorrect?
    A. Implied volatility provides an understanding of the investors' opinions on volatility of the underlying.
    B. If an option's implied volatility is higher than an investor's volatility expectations, the investor will consider the option to be overvalued.
    C. Implied volatility is not comparable for options with different exercise prices and expirations.

Solutions

1. A is correct. The multi-period binomial model can be used to value both path-independent and path-dependent options. Section 3.

2. C is correct. A long call option for a single-period is equal to owning 'h' units of partially financed stock. The financed amount is: PV($-hS^- + c^-$), or using the per period risk-free rate, ($-hS^- + c^-$)/(1+ r). Section 3.1.

3. B is correct. For a long put position, the underlying is sold short and a portion of the proceeds is lent. To hedge a long put position h units of the underlying asset S are traded. Section 3.1.

4. B is correct. The hedge ratio = $\frac{p^+ - p^-}{S^+ - S^-}$. $S^+ = 125$, $S^- = 69$. $p^+ = Max(0{,}100 - 125) = 0$. $p^- = Max(0{,}100 - 69) = 31$ Hence h = $\frac{0-31}{125-69} = -0.554$. Section 3.1.

5. A is correct. At T=2, $c^{++} = Max(0, u^2S - X) = Max\ [0, 156.25 - 100] = 56.25$
   $c^{-+} = c^{+-} = Max(0, udS - X) = Max[0, 100 - 100] = 0$
   $c^{--} = Max(0, d^2S - X) = Max[0, 64 - 100] = 0$
   c = PV[E(c2)] = PV[$\pi^2 c^{++} + 2\pi(1-\pi)c^{+-} + (1-\pi)^2 c^{--}$] = $1/(1.03)^2\ [0.51^2 56.25 + 2 \times 0.51 \times 0.49 \times 0 + 0.49^2 0] = 13.7908$. Section 3.2.

6. B is correct. The put option value can be computed simply by applying put–call parity: p = c + PV(X) – S = 14 + 100/$(1 + 0.03)^2$ – 100 = 8.259. Thus, the current put price is €8.26. Section 3.2.

7. B is correct. At T = 1, when a down move occurs because of early exercise option (American-style) p = 100 – 80 = 20.00 instead of 17.1262. Hence the value of put at T = 0 = 9.51. The early exercise premium = $\frac{1}{(1.03)} \times [0.51 \times 0 + 0.49 \times 20] - 8.15 = 9.51 - 8.15 = 1.36$. Section 3.2.

8. B is correct. Using the binomial model, it can be shown that the arbitrage-free value of the call option is $7. Since the option is trading for $8, it is overpriced. An arbitrage profit can be made by selling the overpriced options and buying an appropriate number of underlying shares. Section 3.1.

9. B is correct. The underlying instrument for interest rate options is the spot rate. Section 3.3.

10. A is correct. "Option valuation follows the expectations approach taken one period at a time." B & C are incorrect. A put option on interest rates will be in the money when the exercise rate is above the spot rate, and for in the money call option the spot rate will be above the exercise rate. Section 3.3.

11. A is correct. c+ = $PV_{1,2}[\pi c^{++} + (1-\pi)c^{+-}] = 0.9626\ [0.5 \times 0.00717 + (1 - 0.5)0.0] = 0.003451$. $c^- = PV_{1,2}[\pi c^{+-} + (1-\pi)c^{--}] = 0.9750\ [0.5 \times 0.0 + (1 - 0.5)0] = 0.0$. Section 3.3.

12. C is correct. At T = 0, $c = PV_{rf,0,1}[\pi c^{+} + (1 - \pi)c^{-}] = 0.9705[0.5 \times 0.003451 + (1 - 0.5)0.0] = 0.001675$. Multiplying by 1,000,000 gives 1,675 which is approximately €1,700. Section 3.3.

13. B is correct. The value of a call option can be described as the present value of the expected terminal option payoffs where the discount rate is the risk-free rate and the expectation is based on the risk neutral probability. Section 3.2.

14. C is correct. BSM assumes that short selling *is* allowed. Options A and B are assumptions of BSM. Section 4.2.

15. B is correct. The BSM model has two components: the stock component and the bond component. The stock component is given by $SN(d_1)$ and the bond component is $e^{-rT}XN(d_2)$. The call value is given by the stock component minus the bond component. The put value based on the BSM model is the bond component - $e^{-rT}XN(-d_2)$ minus the stock component - $SN(-d_1)$. Section 4.3.

16. B is correct. The BSM model put value is equal to the cost of a portfolio of bonds bought with proceeds from short selling of the underlying. A & C are correct statements. Section 4.3.

17. B is correct. The spot price of the underlying is S$60. The exercise price is S$62. The expiration is 0.25 years (three months). The risk-free rate is 0.0174. The dividend yield is 0.0035. The volatility is 0.30. Section 4.3.

18. A is correct. The underlying is the spot FX price of 117.60 PKR/€. The risk-free rate is the Pakistani rate, 6.00%, and the carry rate is the European rate of 1.00%. Section 4.3.

19. A is correct. Black's model for call options can be expressed as $c = e^{-rT}[F_0(T)N(d_1) - XN(d_2)]$, where $F_0(T)$ = the futures price at Time 0 that expires at Time T = 6,702, X = exercise price = 6,690. Section 5.1.

20. C is correct. The underlying rate is the FRA rate that expires in one year = 1.20%. Section 5.2.

21. B is correct. For receiver swaptions, the swap component is $(AP)PVA(R_{FIX})N(-d1)$ and the bond component is $(AP)PVA(R_X)N(-d2)$. The receiver swaption model value is simply the bond component minus the swap component. The payer swaption model value is the swap component (AP)PVA(RFIX)N(d1) minus the bond component (AP)PVA(RX)N(d2). Section 5.3.

22. C is correct. The vega of an option is always positive as an increase in volatility, leads to an increase in the call and put option values. A & B are correct statements. Sections 6.1, 6.2, 6.4.

23. B is correct. If the portfolio consists of put options, hedging will involve buying shares, not shorting. Section 6.1.

24. B is correct. To arrive at a delta neutral portfolio N_H = - Portfolio delta / $Delta_H$ = -2,500/0.5 = -5,000 = selling 5,000 call options. Section 6.1.

25. C is correct. Gamma risk arises when there is large jump in the value of the underlying. Gamma measures “the risk that remains once the portfolio is delta neutral” hence A is incorrect. Gamma has the largest value when the option nears at the money, hence B is incorrect. Section 6.2.

26. C is correct. Implied volatility can be used to compare the value of different options with different exercise prices and expirations. A & B are correct statements. Section 6.6.